# Proceedings

of the Forty-Third Annual Meeting of the
North American Chapter of the International Group
for the Psychology of Mathematics Education

## PME-NA 43

October 14 - 17, 2021

Philadelphia

## Productive Struggle

Persevering Through Challenges

Editors: Dana Olanoff, Kim Johnson, & Sandy Spitzer

# Proceedings of the Forty-Third Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education

# *Productive Struggle: Persevering Through Challenges*

## Philadelphia, Pennsylvania, USA
## October 14-17, 2021

**Editors**

Dana Olanoff

*Widener University*

dolanoff@widener.edu

Kim Johnson

*West Chester University*

Kjohnson2@wcupa.edu

Sandy M. Spitzer

*Towson University*

sspitzer@towson.edu

Olanoff, D., Johnson, K., & Spitzer, S.M. (2021). *Proceedings of the forty-third annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Philadelphia, PA.

## Citation

Olanoff, D., Johnson, K., & Spitzer, S. M. (2021). *Proceedings of the forty-third annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Philadelphia, PA.

## Copyright



## Land Acknowledgment

This border around what is colonially known as Pennsylvania represents a tragic and unjust history. We acknowledge the Lenape, Munsee, Susquehannock, Osage, Erie, Massawomeck and Haudenosaunee Tribes, among others, on whose ancient and sacred land we hold this conference. As a PME-NA community we recognize the ever-present systemic inequities that stem directly from past wrongdoings, and we commit ourselves indefinitely to respecting and reconciling this long history of injustice.

## PME-NA History and Goals

PME came into existence at the Third International Congress on Mathematical Education (ICME-3) in Karlsrühe, Germany, in 1976. It is affiliated with the International Commission for Mathematical Instruction. PME-NA is the North American Chapter of PME. The first PME-NA conference was held in Evanston, Illinois in 1979. Since their origins, PME and PME-NA have expanded and continue to expand beyond their psychologically-oriented foundations.
The major goals of the International Group and the North American Chapter are:

1. To promote international contacts and the exchange of scientific information in the psychology of mathematics education;
2. To promote and stimulate interdisciplinary research in the aforesaid area, with the cooperation of psychologists, mathematicians, and mathematics teachers; and
3. To further a deeper and better understanding of the psychological aspects of teaching and learning mathematics and the implications thereof.

## PME-NA Membership

Membership is open to people who are involved in active research consistent with PME-NA's aims or who are professionally interested in the results of such research. Membership is open on an annual basis and depends on payment of dues for the current year. Membership fees for PME-NA (but not PME International) are included in the conference fee each year. If you are unable to attend the conference but want to join or renew your membership, go to the PME-NA website at http://pmena.org. For information about membership in PME, go to http://www.igpme.org and visit the "Membership" page.

Olanoff, D., Johnson, K., & Spitzer, S.M. (2021). *Proceedings of the forty-third annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Philadelphia, PA.

## PME-NA Steering Committee

**Elected Members**
Julie Amador (chair), *University of Idaho,* jamador@uidaho.edu
José Martínez Hinestroza, *Texas State University*
Alyson E. Lischka, *Middle Tennessee State University*
Jennifer Holm, *Wilfrid Laurier University*
Oyemolade (Molade) Osibodu, *York University*
Ana Isabel Sacristán, *Center for Research and Advanced Studies (Cinvestav)*
María S. García González, *Universidad Autónoma de Guerrero*
Karl Kosko, *Kent State University*
Jessica Hunt, *North Carolina State University*

**Appointed Members**
Aaron Brakoniecki (webmaster), *Boston University*, brak@bu.edu
Ji-Yeong I (treasurer), *Iowa State University*, jiyeongi@iastate.edu

## Conference Chairs

### Past Conference Co-Chairs (2018–2021)

Ana Isabel Sacristán
*Center for Research and Advanced Studies (Cinvestav)*, Mexico
asacrist@cinvestav.mx

José Carlos Cortés-Zavala
*AMIUTEM / Universidad Michoacana de San Nicolaìs de Hidalgo*, Mexico
jcortes@umich.mx

### Current Conference Co-Chairs (2019–2022)

Sandy Spitzer
*Towson University*
sspitzer@towson.edu

Dana Olanoff
*Widener University*
dolanoff@widener.edu

Kim Johnson
*West Chester University of Pennsylvania*
Kjohnson2@wcupa.edu

### Future Conference Co-Chairs (2020–2023)

Alyson Lischka
*Middle Tennessee State University*
Alyson.Lischka@mtsu.edu

Jeremy Strayer
*Middle Tennessee State University*
jeremy.strayer@mtsu.edu

Elizabeth Dyer
*University of Tennessee Knoxville*
edyer8@utk.edu

Jennifer Lovett
*Middle Tennessee State University*
Jennifer.Lovett@mtsu.edu

Ryan Seth Jones
*Middle Tennessee State University*
Ryan.Jones@mtsu.edu

Olanoff, D., Johnson, K., & Spitzer, S.M. (2021). *Proceedings of the forty-third annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Philadelphia, PA.

## 2021 PME-NA Local Organizing Committee

Sandy Spitzer, Towson University (Chair)
Dana Olanoff, Widener University (Co-Chair)
Kim Johnson, West Chester University (Co-Chair)

Kimberly Corum, Towson University (Technology Coordinator)

Graduate Assistants:
Rachael Talbert, Towson University
Kayla Begen, Towson University
Sarah Gill, West Chester University

## Volunteers

The Local Organizing Committee is very grateful to the team of volunteers who helped make the conference possible and facilitated interactions between in-person and virtual participants.

### Conference Volunteers

Ruby Ellis, *North Carolina State University*
Allison Gantt, *University of Delaware*
Beth MacDonald, *Utah State University*
Matthew Melville, *University of Delaware*
Rachael Talbert, *Towson University*
Karen Zwanch, *Oklahoma State University*

### Discussion Session Leaders

Ayman Aljarrah, *Acadia University*
Kimberly Corum & Rachael Talbert, *Towson University*
Carlos Nicolas Gomez Marchant & Stacy R. Jones, *University of Texas*
Crystal Kalinec-Craig, *University of Texas-San Antonio*
Alexa W.C. Lee-Hassan, *University of Illinois Chicago*
Samuel Otten, *University of Missouri*
Sam Prough, *University of Delaware*
Amanda Reinsburrow, *Drexel University*
Rachel Tremaine, *Colorado State University*
Bima Sapkota, *Purdue University*
Caro Williams-Pierce, *University of Maryland*

**Session Facilitators**

Denish Akuom, *Montclair State University*
Nigar Altindis, *University of New Hampshire*
Carrie Bala, *Utah State University*
Erin Baldinger, *University of Minnesota*
Mona Baniahmadi, *Duquesne University*
Brianna Bentley, *North Carolina State University*
Steven Boyce, *Portland State University*
Lori Burch, *Indiana University*
Rob Ely, *University of Idaho*
Nicole M. Wessman-Enzinger, *George Fox University*
Ben Freeburn, *Western Michigan University*
Julia St. Goar, *Merrimack College*
Steven Greenstein, *Montclair State University*
Lisa Hawley, *Michigan State University*
Jose Martinez Hinestroza, *Texas State University*
Jenifer Hummer, *West Chester University of Pennsylvania*
Michelle Morgan King, *Western Colorado University*
Keith Leatham, *Brigham Young University*
Beth MacDonald, *Utah State University*
Allison McCulloch, *University of North Carolina at Charlotte*
Alesia Mickle Moldavan, *Fordham University*
Chelsea Muhs, *Portland State University*
Nirmala Naresh, *University of North Texas*
Olanrewaju Oriowo, *University of North Carolina at Charlotte*
Blake Peterson, *Brigham Young University*
Luke Reinke, *University of North Carolina at Charlotte*
Gizem Solmaz, *Florida State University*
Megan Staples, *University of Connecticut*
Irma Stevens, *University of Michigan*
Shari Stockero, *Michigan Technological Universit*y
Travis Weiland, *University of Houston*
Xiangquan Yao, *The Pennsylvania State University*
Karen Zwanch, *Oklahoma State University*

## Strand Leaders

The Local Organizing Committee is extremely appreciative of the following people for serving as Strand Leaders. They managed the reviewing process for their strand and made recommendations to the Local Organizing Committee. The conference would not have been possible without their efforts.

Neet Priya Bajwa, *Illinois State University*
Pavneet Kaur Bharaj, *Indiana University Bloomington*
Brian Bowen, *West Chester University of Pennsylvania*
Charity Cayton, *East Carolina University*
Lara Dick, *Bucknell University*
Heather Gallivan, *Northern Iowa University*
María S. García González, *Universidad Autónoma de Guerrero*
Maureen Grady, *East Carolina University*
Duane Graysay, *Syracuse University*
José Martínez Hinestroza, *Texas State University*
Charles Hohensee, *University of Delaware*
Melike Kara, *Towson University*
Shiv Karunakaran, *Michigan State University*
Monica Karunakaran¸ *Michigan State University*
Signe Kastberg, *Purdue University*
Erika Litke, *University of Delaware*
Valerie Long, *Indiana University of Pennsylvania*
Jennifer Lovett, *Middle Tennessee State University*
Sararose Lynch, *Westminster College*
Marta Magiera, *Marquette University*
Allison McCulloch, *University of North Carolina at Charlotte*
Susanna Molitoris Miller, *Kennesaw State University*
Emily Miller, *West Chester University of Pennsylvania*
Cody Patterson, *Texas State University*
Christine Phelps-Gregory, *Central Michigan University*
Priya Prasad, *University of Texas – San Antonio*
Patrick Sullivan, *Missouri State University*
Barbara Swartz, *West Chester University of Pennsylvania*
Jennifer Ward, *Kennesaw State University*
Robert Wieman, *Rowan University*
Xiangquan Yao, *Pennsylvania State University*

## Reviewers

Likewise, the Local Organizing Committee is also very appreciative of the following colleagues for peer-reviewing submissions to the conference:

Abbaspour Tazehkand, Shahabeddin
Acevedo, Carlos Ivan
Acuña, Claudia Margarita
Aguilar, Jair
Ahrens, Sally
Akuom, Denish Ogweno
Alapala, Burcu
Aljarrah, Ayman
Altindis, Nigar
Altshuler, Mari
Alyami, Hanan
Al-Younes, Mohammad Saleh
Alzaga Elizondo, Tenchita
Amador, Julie
Amidon, Joel
Amman, Kristen
Anantharajan, Madhuvanti
Anderson, Kiera
Anderson, Robin Keturah
Andersson, Annica
Andreasen, Janet
Andresen, Mette
Anthony, Monica
Antonides, Joseph
Arellano, Beatriz
Armstrong, Alayne
Arnold, Elizabeth G
Arslan, Zeynep
Asempapa, Reuben
Ataide Pinheiro, Weverton
Atanga, Napthalin Achubang
Austin, Christine Kathryn
Avila Zarate, Adriana Ines
Azimi Asmaroud, Seyedehkhadijeh
Azmy, Christina
Bae, Younggon
Bailey, Nina Gabrielle
Baker, Katherine
Bala, Carrie Olson
Baldinger, Erin E.
Baniahmadi, Mona
Barabe, Genevieve
Barno, Erin
Belcher, Michael
Bell, Amanda
Bennett, Amy Been
Benoit, Gregory
Bentley, Brianna
Bergman, Anna Marie
Bertolone-Smith, Claudia Marie
Bharaj, Pavneet Kaur
Bianco, Kathryn
Billings, Esther M
Bishop, Jessica Pierson
Black, Matthew
Bock, Camden Glenn
Bofferding, Laura
Boileau, Nicolas
Bondurant, Liza
Bostic, Jonathan David
Boyce, Steven
Brady, Corey
Brass, Amy
Brewer, Jacob
Brown, David
Brown, Rachael Eriksen
Brunner, Megan
Bulut, Gamze
Buntin, Charity
Burch, Lori
Busi, Rich
Butler, Rebecca
Callard, Cynthia H
Calleros, Ernesto D
Campbell, Matthew P
Capozzoli, Michelle
Carman, Luke B
Carney, Michele
Carson, Cynthia
Cavanna, Jillian M
Cavey, Laurie Overman
Cayton-Hodges, Gabrielle Alexis
Çelik, Derya
Cevik, Emel
Champion, Joe
Chao, Theodore
Chen, Lizhen
Cheng, Peter
Cho, Youngkee
Cirillo, Michelle
Closser, Avery Harrison
Condon, Lara
Conner, Annamarie
Conner, Kimberly
Contreras, Jose
Cordero-Siy, Eric
Corey, Douglas
Correa, Priscila D
Corum, Kimberly
Corven, Julien
Cox, Jennifer Lynn
Cox, Wesley A
Crawford, Angela R
Crawford-Ferre, Heather
Cui, Xiaowen
Czap, Lindsay Nicole
Czocher, Jennifer A
Dahl, Bettina
Dames, Brendan
Davis, Joy Anderson
Dawkins, Paul Christian
De Araujo, Zandra
Debay, Dennis J
Dejarnette, Anna Fricano
Delaney, Victoria
Desai, Siddhi
Dexter Torti, Cameron John
Dickson, Chelsea
Dietiker, Leslie
Dimmel, Justin
Dinapoli, Joseph
Dışbudak Kuru, Özge
Dobie, Tracy Elyse
Dogan, Muhammed Fatih
Donaldson, Sara

Kirwan, J Vince
Klein, Valerie
Kline, Kate
Knapp, Melinda
Ko, Yi-Yin
Kocabas, Sezai
Koehne, Christina
Koestler, Courtney
Konuk, Nursen
Kopparla, Mahati
Kosko, Karl Wesley
Krause, Gladys
Krejci, Brooke
Kronenberg, Deborah
Kruger, Jennifer
Krupa, Erin E.
Küchle, Valentin Alexander Balthasar
Kwon, Minsung
Lai, Yvonne
Lambert, Rachel
Lang, Julie
Larison, Sarah
Leatham, Keith R.
Lee, Alees
Lee, Carrie W
Lee, Hea-Jin
Lee, Hollylynne S
Lee, Hyunjeong
Lee, Inyoung
Lee, Yi-Jung
Lee-Hassan, Alexa W.C.
Leonard, Jacqueline
Leshin, Miriam S
Lewis, Hannah
Libberton, Jason
Lischka, Alyson E.
Liu, Jinqing
Lo, Jane Jane
Lolkus, Michael
Louie, Josephine
Macarthur, Kelly (She/Her)
Macdonald, Beth L.
Machalow, Rowan
Maclean, Mark
Magiera, Marta T.
Mamolo, Ami
Mannix, Joshua P
Mansour, Rue
Margolis, Claudine
Marsh, Dalton Dayne
Martin, Stephanie
Martinez Hinestroza, Jose
Martinez, Antonio Estevan
Marynowski, Richelle
Mask, Walker
Matranga, Anthony
Max, Brooke
Mbewe, Rose
Mcfeetors, P. Janelle
Mcgraw, Rebecca
Meagher, Michael S
Membreño Estrada, Sharon Samantha
Mitchell, Rebecca
Mkhatshwa, Thembinkosi Peter
Moldavan, Alesia Mickle
Molitoris Miller, Susanna
Monarrez, Angelica
Monroe, Ann
Montero-Moguel, Luis Emmanuel
Moore, Alexander
Moore-Russo, Deborah
Morgado, Cindy Nathalia
Morrissey, Susie
Morrow-Leong, Kimberly
Morton, Karisma
Moss, Diana L.
Muhs, Chelsea A
Myers, Kayla
Mynatt, Gabrielle Elizabeth
Nagle, Courtney
Naresh, Nirmala
Navarro Robles, María Estela
Neumayer Depiper, Jill
Newton, Jill
Ngo, Vy Vo Hoang
Nguyen, Phi
Nikula, Johannah
Nirode, Wayne
Norris-Leblanc, Chris
Nti-Asante, Emmanuel
O'bryan, Alan
Öçal, Mehmet Fatih
Olarte, Royce
Olson, Gary
Oriowo, Olanrewaju
Orrill, Chandra Hawley
Otten, Samuel
Ozturk, Ayse
Ozturk, Nejla
Palisse, Jennifer
Pandiscio, Eric
Panorkou, Nicole
Paoletti, Teo
Park, Hyejin
Park, Joo Young
Park, Sunyoung
Parks, Amy Noelle
Parr, Erika David
Pelaez, Kevin
Perry, Jill A
Piatek-Jimenez, Katrina
Pinter, Holly Henderson
Popovic, Gorjana
Powell, Sarah
Preciado Babb, Armando Paulino
Prough, Sam
Przybyla-Kuchek, Julia
Rahman, Zareen Gul
Raja, Waleed Ashraf
Rapke, Tina
Rasmussen, Chris
Reedman, Emma
Remillard, Janine
Rhine, Steve
Ricks, Thomas
Ristroph, Ingrid
Rivera, Seema
Roberts, Sarah A.
Roberts, Thomas
Robinson, Molly L
Rodriguez, Jon-Marc Gregory
Roehrig, Amy E
Roh, Kyeong Hah
Roman, Christopher Orlando
Rosencrans, Brenda
Rotem, Sigal Hava
Ruff, Adam
Runnalls, Cristina
Rupnow, Rachel

Ruttenberg-Rozen, Robyn
Sabree, Minnah Jirani
Saclarides, Evthokia Stephanie
Safak, Reyhan
Safi, Farshid
Sahmbi, Gurpreet
Saini, Akash Kumar
Saldaña, Mike
Salter, Diana G
Sampson, Stéphanie
Sánchez, Ernesto Alonso
Sankaranarayanan, Ananthi
Santana, Paula
Sanz, Maria Teresa
Sapkota, Bima Kumari
Satyam, V. Rani
Savich, Theodore Michael
Schack, Edna O'brien
Scharfenberger, Adam
Schoen, Robert
Schraeder, Nurul Wahyuni
Scott, Mallika H
Seat, Jennifer
Seiwell, Amanda
Sencindiver, Benjamin D.
Shah, Fahmil
Sharpe, Sheree
Shaver, Nick
Shumway, Jessica
Sianturi, Iwan Andi Jonri
Siebert, Daniel
Simon, Sarina
Singh, Rashmi
Smith, Erin
Smith, Ethan
Smith, James
Smith, Shawnda
Smucker, Karoline
Solmaz, Gizem
Sorge, Brandon
Soto, Hortensia
Spiteri, Andrew
Spitzer, Sandy M
Stephan, Michelle
Sternberg, Kateri A
Stevens, Alexis
Stevens, Irma
Stockero, Shari L
Stoddard, Elyssa
Stoehr, Kathleen Jablon
Street, Ciera
Strickland, Sharon
Sturgill, Derek
Suárez, Mayra Zulay
Suazo-Flores, Elizabeth
Sundrani, Anita
Sung, Hanall
Swars Auslander, Susan
Swartz, Micah
Tague, Jenna
Tanguay, Carla Lynn
Taub, Michelle
Taylor, Cynthia E.
Tchoshanov, Mourat
Teuscher, Dawn
Thacker, Ian
Thompson, Jennifer
Tillema, Erik S
Totorica, Tatia Baum
Turner, Kyle Russell
Tyburski, Brady A
Udun, Yalcin
Uscanga, Rosaura
Uysal, Seyda
Vahle, Courtney
Valenzuela, Carlos
Valerio, Jennifer Lynn
Valero, Jonathan Rojas
Van De Sande, Carla
Van Zoest, Laura
Vandenberg, Jana Elle
Vargas-Alejo, Veronica
Villafañe-Cepeda, Wanda
Vroom, Kristen
Walker, William
Walsh, Patricia A
Wambua, Mitchelle Mbete
Wang, Xiong
Ward, Jennifer
Warshauer, Hiroko Kawaguchi
Waswa, Anne Nyarotso
Watford, Mark
Webel, Corey
Weiland, Travis
Weinberg, Aaron
Wessman-Enzinger, Nicole Marie
Westby, Kathryn R.
Wheeler, Ann
Whitehead, Ashley
Wieman, Rob
Wrightsman, Elizabeth
Wynn, Lynda
Xia, Fangli
Yalman Ozen, Demet
Yan, Xiaoheng
Yang, Xiaotong
Yao, Xiangquan
Yeo, Sheunghyun
Ying, Yufeng
Zahner, William
Zarkh, Anna
Zbiek, Rose Mary
Zelkowski, Jeremy
Zhang, Jing
Zhang, Pingping
Zhou, Lili
Zolfaghari, Maryam
Zuniga Ruiz, Sandra
Zwanch, Karen

# Preface

Dear Colleagues,

On behalf of the 2021 PME-NA Steering Committee, the PME-NA 43 Local Organizing Committee, Towson University, Widener University, and the West Chester University of Pennsylvania, we welcome you to Philadelphia, Pennsylvania, USA, for the Forty-Third Annual Meeting of the International Group for the Psychology of Mathematics Education – North American Chapter, held at the Sheraton Philadelphia Downton and virtually.

This year's conference theme is **Productive Struggle: Persevering through Challenges**. The years of 2020-2021 brought a global pandemic and with it, many challenges for mathematics education research. Public schools faced a sudden and prolonged transition to distance education, while higher education experienced a budget crisis as well as the loss of in-person classes and traditional field experiences for teacher education. Many researchers and their communities have encountered unforeseen difficulties including personal or family illness, employment loss, and dramatically increased caregiving responsibilities, all of which fell disproportionately onto already-vulnerable populations. Meanwhile, demonstrations for racial justice highlighted the insidious effects of racism throughout our society. All these challenges reflect long-term issues, while highlighting and uncovering the effects of centuries of unjust structures and systems.

By choosing the theme of *persevering through challenges*, and in Philadelphia, a city which has historically represented an optimistic spirit and a belief in a better tomorrow, we aim to encapsulate an idea of hope towards the future: that through struggle, and through scholarly work, engagement in our community, and sustained effort towards improvement, we can truly make a difference in the lives of teachers and students, and in mathematics education broadly in continent of North America.

We hope this conference serves to provoke learning through productive struggle and to support our field in persevering through these continuing challenges in mathematics education. In particular, we hope that this conference can serve as a model and precedent for implementing a hybrid research conference. Early in the process of planning PME-NA43, we committed to the idea of a fully hybrid conference with the guiding principle that all opportunities should be equally available to both in-person and virtual participants. Each of the 3 plenary talks, 15 working groups or research colloquia, 160 research sessions (presenting a total of 239 papers), and 121 poster presentations are available for live participation and interaction between in-person and virtual participants.

This year's conference will be attended (either in-person or virtually) by more than 640 researchers, faculty members, and graduate students from around the world including Canada, Mexico, Australia, Israel, Cameroon, and across the USA. Each paper was reviewed by multiple referees in an anonymous review process. The result was an overall acceptance rate of 79% of papers accepted in some form (not necessarily in the form in which they were submitted), with 37% of research report submissions accepted as research reports, 48% of brief research report

submissions accepted as brief research reports, 83% of poster submission accepted as posters, and 94% of working group submissions accepted. The papers eventually accepted comprised 81 research reports, 158 brief research reports, 121 Posters, and 15 Working Groups or Research Colloquia.

For this conference we created new strands and reframed others. Most notably, we reconfigured the mathematics content strands from being organized by content area (e.g. Geometry, Algebra, Number Concepts) to being organized by grade band (Elementary/Middle Years, comprising early childhood, elementary, and middle-grades mathematics topics; and Later Years, comprising secondary and post-secondary topics.)

We thank the many people who generously volunteered their time over the past year in preparation for this conference. In particular, we thank the three graduate assistants who contributed to these proceedings: Rachael Talbert (Towson University), Kayla Begen (Towson University), and Sarah Gill (West Chester University). Thanks to Carly Sullivan for her invaluable support in planning the in-person events. We would specifically like to highlight the herculean efforts of Kimberly Corum (Towson University) in developing the online conference hub.

We hope that the papers presented within these Proceedings will give you engaging, inspiring, and challenging ideas to transform your practice. And, as we continue to endure a time of challenge and struggle across North America, we hope that this conference can be a learning opportunity for the field to think about what it means to be an active and engaged professional, and how the structure of conferences can support faculty and students across many stages of their lives and careers in *persevering through challenges*.

Thank you,

The PME-NA43 Local Organizing Committee

Dana Olanoff
Widener University
dolanoff@widener.edu

Sandy Spitzer
Towson University
sspitzer@towson.edu

Kim Johnson
West Chester University
kjohnson2@wcupa.edu



# Contents



Olanoff, D., Johnson, K., & Spitzer, S. M. (2021). *Proceedings of the forty-third Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Philadelphia, PA.

# ATTENDING TO AIMS IN ALGEBRA: THE JUGGLE STRUGGLE

Yufeng Ying
yufengsmart@gmail.com

Brandon K. Singleton
University of Georgia
bksingleton@uga.edu

*In this report, we explore the nature of aims for algebra instruction. First, we examine the major aims that have informed algebra education and curriculum reform from the 1960s into the current era. The relationships between aims are marked by compatibility as well as tension. We argue for researching and viewing aims as enacted priorities that are revealed through the everyday choices algebra educators make.*

Keywords: Algebra and Algebraic Thinking, Curriculum, Research Methods, Teacher Beliefs

Algebra is a versatile subject. Scholars argue that algebra fosters generalization (Usiskin, 1995) and the recognition and use of structures (Kieran 1989). Algebra instruction can raise students' awareness of social injustice (Gutstein, 2006) and encourage autonomy (Kosko, 2016) and creativity (Chiu, 2008). However, the multiplicity of aims for algebra education can also bring a real challenge to today's algebra teachers: How should we coordinate the many aims for algebra learning? Should teachers organize their teaching with an equal emphasis on all possible aims that algebra education can carry? How do teachers make decisions on the aims that they pursue? The purpose in this paper is to discuss the need to identify and coordinate the many aims for algebra education.

## Four Enduring Curriculum Aims

It is helpful to organize aims into a framework that captures the most central and enduring purposes for teaching algebra. One useful framework was developed by Kliebard (2004) while characterizing education during the early twentieth century. Kliebard proposed four major groups: humanists, developmentalists, social efficiency proponents, and social meliorists.

1. Humanists cherished western cultural heritage and the disciplinary value of classical subjects that increase students' mental power.
2. Developmentalists believed that the natural cognitive or psychological development of children should be given first priority when determining teaching content.
3. Social efficiency proponents concerned themselves with the needs of schools in a rapidly changing society, turning to the standardized techniques of industry and business.
4. Social meliorists believed education should actively foster social equity.

These four categories offer a means to explore the aims mathematics educators have emphasized throughout distinct historical periods.

During the 1960s, an array of curriculum projects known as the New Math (Phillips, 2014) took place in which mathematicians sought to ground school mathematics in the structure of the discipline. For instance, algebra during the new math movement was taught as an axiomatic system. Educators emphasized the importance of revealing the inherent and hidden structures behind algebra, such as set theory and concepts from abstract algebra (Herrera & Owens, 2001). Because this curriculum trend was led by mathematicians and emphasized the disciplinary value

of mathematics, we might regard their aims during this period as in line with humanists.

By the 1980s, dissatisfaction with New Math made space for alternatives. Constructivism was one important response. Constructivists (e.g., Steffe & Kieren, 1994; Confrey, 1990; Ernest, 1994) felt that the psychological realities of young children, rather than the professional norms of mathematics as a discipline, dictated the aims of teaching and learning mathematics. Through research into student thinking and learning, algebra teachers started to recognize a significant gap between formal mathematics and students' own experiences with mathematics, and topics such as the transition from arithmetic to algebra gained attention in the field (e.g., Filloy & Rojano, 1989; Schoenfeld & Arcavi, 1988). Inspired by prominent psychologists such as Piaget and Dewey, constructivism can be characterized as a shift from the earlier humanist approach toward a developmentalist approach.

In 1983, the National Commission on Excellence in Education published a report titled *A Nation At Risk*, using an alarmist tone to bring national attention to perceived weaknesses in the American education system. The report's galvanizing influence formed the backdrop for efforts to create standards, measurement tools, and accountability policies for systemic educational improvement. These efforts targeted the efficiency of mathematics education as a system, working to cultivate mathematical knowledge as widely and effectively as possible. In the pursuit of efficiency through standardization and accountability, other educational aims were sometimes pushed aside when achievement was used as the indicator of national prosperity (Berliner, 2011). Teachers in algebra classes have felt pressured by the need for test preparation, adopting pedagogies with certain compromises and sacrifices (see Gutstein, 2006).

An ongoing movement in mathematics education that can be associated with aims of social meliorists is known as the "sociopolitical turn" (Gutierrez, 2013). In recent years, an increasing number of socially-minded mathematics educators proposed that teachers of mathematics should use their instruction to take part in solving social problems to create a more equitable society. In algebra education, scholars have promoted more culturally relevant pedagogies and equity-centered problem-solving approaches in teaching (e.g., Ligocki, 2017; Boaler & Sengupta-Irving, 2016; Gutstein, 2006).

The history of algebra education suggests that at a broad level, aims differ, aims can rise and fall, compete for attention, and overlap in complicated ways.

## Aims and Priorities

The overview above suggests that there has not been a single, uniform idea about what constitutes "good algebra." Rather, educators' visions of algebra education have fluctuated throughout history in response to different but persisting educational aims. How then should educators navigate the existence of different aims?

We argue the first step in navigating aims is to think of aims as potential priorities. Priorities are objectives that require intentional effort. Consequently, the tension between aims plays out in a subtle dynamic as educators at all levels make countless choices about what should be taught and how. Therefore, the struggle between aims is not only an ideological debate but is also a practical challenge that algebra educators face every day.

To illustrate such a point in more detail, we explore one form of tension in algebra teaching between two fabricated teachers named Jack and Rose. Both teachers are preparing lessons with the main purpose of helping students to become familiar with multiplication.

Jack graduated with a master's degree in mathematics education and enjoyed reading research about students' algebraic conceptions. Therefore, in preparing the lesson, Jack decides to mimic an activity that Kaput (1999) highly praised, where the teacher helped students to informally

prove the commutative property of multiplication by using arrays of sticks. Jack structures the lesson by planning to first ask students to use arrays of sticks to represent the products of different integers, such as $4 \times 7$. He expects students will likely generate at least two ways of representing the product (4 rows of 7, and 7 rows of 4). Jack will leverage those activities and invite students to think about whether different representations will have different total numbers of sticks. Then, Jack may guide students to realize that reversing the order of multiplication is exactly like rotating the number of rows with the number of columns for arrays of sticks. Since transposing rows with columns does not change the total number of sticks, changing the order of multiplication should preserve the product. In general, Jack may hope the students can both practice multiplication through this project and engage in other desired mathematical activities such as generalizing and creating mathematical representations.

Rose also graduated with a master's degree in mathematics education and enjoyed reading research about equity in mathematics classrooms. Therefore, in preparing her lesson, Rose decides to create a mini social project similar to what Gutstein (2006) has shared. Rose selects water consumption as the central issue. Rose may start to provide students a list of common water-consuming activities along with the average water use of each and ask students to first decide the gallons of water they think are needed for an average person or family per day. (For example, washing one's hands uses 2 gallons of water, so a person who washes his or her hands 4 times per day requires 8 gallons in total.) Then, Rose may provide students with information about how different nations have different average rates of water consumption per individual and ask students to calculate an average person's possible water-consuming activities depending on the country of residence. Through careful sequencing and structuring, Rose hopes that the students not only complete a list of multiplication problems but also use the results of their multiplication to have a broad understanding of the international inequality of water consumption and develop good habits of conserving water.

Jack and Rose may or may not know about any theoretical categories of aims describing their choices. Still, consciously or unconsciously, every pedagogical choice that Jack and Rose have adopted is also a choice between different educational priorities and aligns with different educational aims. Indeed, research on teacher beliefs has widely reported the following: a) teachers develop a complicated set of values and beliefs; b) those values and beliefs guide and influence their everyday teaching, planning, and assessment; c) those values and beliefs frequently do not need evidence to back them up; and d) direct training in certain pedagogical models shifts teachers' beliefs and principles (e.g., Rimm-Kaufman et al., 2006; Kagan, 1992; Richardson, 1996). Building from this literature, we argue that different educational aims act as distinct educational priorities that influence almost every instance of small or large decision making throughout the educational process. Teachers face choices between educational priorities when they are picking which task or activity to implement, researchers face choices between educational priorities when they are deciding which topic to research, and administrators face choices between educational priorities when they are judging which curriculum and policy to use.

Such a conceptualization of aims as educational priorities is consistent with our earlier discussion of the historical fluctuation of aims in the algebra curriculum. Researchers and educators who advocate a particular type of aim rarely deny the value of other possible aims. However, they do tend to make an intentional effort to prioritize their own preferred aim over others during research and curriculum reform. Thus, in a sense, aims are commensurable, as one's choice in picking a certain priority does not suggest one's denial of the value of other

priorities. But aims conflict with each other as the options contributing to different aims compete for educators' intentional effort. The question we are left with is how the conceptualization of aims as educational priorities brings new insight into the work of juggling between aims.

## Implications and Recommendations for Future Research

We derive several important implications from conceptualizing aims as priorities. The first implication is that aims are not cost-free and tensions between aims will inevitably persist. If competition between aims is viewed as a philosophical dispute, then aims might be reconciled at a theoretical or ideological level by weaving aims together into some grand, comprehensive quilt. However, situating the juggling of aims as an empirical reality of making choices between educational priorities suggests that tensions will persist and prioritizing aims will always have its costs.

Second, the tension and coordination between aims should be informed by research. All teachers, researchers, and policy makers are constantly picking their own priorities in their decision making and selecting their own preferred aims for their work. Consequently, there is a need to develop theoretical constructs in helping educators in all branches to conceptualize the tension and tradeoff between each aim along with an aim's relative affordances and constraints. To make aims an explicit object of research calls for expanding existing branches of research. Much of mathematics education research can be summarized as design science (see Cobb, 2007) in which teacher-researchers attempt to study and improve mathematics teaching and learning by drawing from various paradigms of scientific inquiry. When researchers conduct design science, they choose aims somewhat freely and they study the settings in which those aims can most profitably be observed and improved. Given an aim that is deemed valuable *a priori*, what are the principles by which to attain it? This research is useful, but we call for new research that adopts a different underlying premise: Given a setting with competing aims at work, what are those aims, where do they originate from, how are they prioritized and negotiated, and what are the consequences or implications of attaining or failing to attain each aim? Such research intentionally surveys and coordinates different aims by addressing the "economy" behind various priorities. For instance, not all aims are equally viable in different content areas or settings. Similarly, some aims can be satisfied with a small amount of intentional effort while others require more. Some aims have broad implications, others do not. Knowing the economy of aims helps researchers and practitioners to prioritize aims via a rigorously informed and justified process. (For interested readers, we recommend Pais (2013), Lundin (2012), and Wagner (2017) as some relevant work.)

The third implication is to respect educators holding different aims. This report does not call for a hierarchical ranking of all aims. Rather, research provides perspective to select aims more clearly. This proposal echoes Rorty's (1979/2009) idea of hermeneutic philosophy and Piaget's (2013) view that a central objective of philosophy is the "coordination between values" (p. 3). Research and scholarship about the aims of algebra education do not function as supreme guidance which teachers ought to follow, but rather as an instructive knowledge base that educators consult when selecting values, setting aims, and working to attain them (see Hiebert, 1999). We respect people’s right to pursue different aims, but just as importantly, we hope every choice can become an increasingly informed and justified choice.

## Reference


Berliner, D. (2011). Rational responses to high stakes testing: The case of curriculum narrowing and the harm that follows. *Cambridge Journal of Education*, *41*(3), 287-302.

Boaler, J., & Sengupta-Irving, T. (2016). The many colors of algebra: The impact of equity focused teaching upon student learning and engagement. *The Journal of Mathematical Behavior*, *41*, 179-190.

Chiu, M. M. (2008). Effects of argumentation on group micro-creativity: Statistical discourse analyses of algebra students' collaborative problem solving. *Contemporary Educational Psychology*, *33*(3), 382-402.

Cobb, P. (2007). Putting philosophy to work: Coping with multiple theoretical perspectives. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 3–38). Information Age.

Confrey, J. (1990). Chapter 8: What constructivism implies for teaching. *Journal for Research in Mathematics Education. Monograph*, *4*, 107-210.

Ernest, P. (1994). Social constructivism and the psychology of mathematics education. *Constructing mathematical knowledge: Epistemology and mathematical education* (Vol. 4, pp. 62-72). RoutledgeFalmer.

Filloy, E., & Rojano, T. (1989). Solving equations: The transition from arithmetic to algebra. *For the Learning of Mathematics*, *9*(2), 19-25.

Gutiérrez, R. (2013). The sociopolitical turn in mathematics education. *Journal for Research in Mathematics Education*, *44*(1), 37-68.

Gutstein, E. (2006). *Reading and writing the world with mathematics: Toward a pedagogy for social justice*. Taylor & Francis.

Herrera, T. A., & Owens, D. T. (2001). The "new new math"?: Two reform movements in mathematics education. *Theory into Practice*, *40*(2), 84-92.

Hiebert, J. (1999). Relationships between research and the NCTM standards. *Journal for Research in Mathematics Education*, *30*, 3-19.

Kagan, D. M. (1992). Implication of research on teacher belief. *Educational Psychologist*, *27*(1), 65-90.

Kaput, J. (1999). Teaching and learning a new algebra with understanding. In E. Fennema & T. Romberg (Eds.), *Mathematics classrooms that promote understanding* (pp. 133-155). Erlbaum.

Kieran, C. (1989). The early learning of algebra: A structural perspective. In S. Wagner & C. Kieran (Eds.), *Research issues in the learning and teaching of algebra* (Vol. 4, pp. 33-56). Erlbaum.

Kliebard, H. M. (2004). *The struggle for the American curriculum, 1893-1958*. Psychology Press.

Kosko, K. W. (2016). Primary teachers' choice of probing questions: Effects of MKT and supporting student autonomy. *International Electronic Journal of Mathematics Education*, *11*(4), 991-1012.

Ligocki, D. T. (2017). Inequalities in math: Using the algebra classroom as a site for social justice. *Journal of Critical Thought and Praxis*, *6*(1).

Lundin, S. (2012). Hating school, loving mathematics: On the ideological function of critique and reform in mathematics education. *Educational Studies in Mathematics*, *80*(1), 73-85.

National Commission on Excellence in Education. (1983). A nation at risk: The imperative for educational reform. *The Elementary School Journal*, *84*(2), 113-130.

Pais, A. (2013). An ideology critique of the use-value of mathematics. *Educational Studies in Mathematics*, *84*(1), 15-34.

Phillips, C. J. (2014). *The new math: A political history.* University of Chicago Press.

Piaget, J. (1971). *Insights and illusions of philosophy* (W. Mays, Trans.). The World Publishing.

Richardson, V. (1996). The role of attitudes and beliefs in learning to teach. *Handbook of research on teacher education*, *2*(102-119), 273-290.

Rimm-Kaufman, S. E., Storm, M. D., Sawyer, B. E., Pianta, R. C., & LaParo, K. M. (2006). The Teacher Belief Q-Sort: A measure of teachers' priorities in relation to disciplinary practices, teaching practices, and beliefs about children. *Journal of School Psychology*, *44*(2), 141-165.

Rorty, R. (2009). *Philosophy and the mirror of nature*. Princeton University Press. (Original work published 1979)

Schoenfeld, A. H., & Arcavi, A. (1988). On the meaning of variable. *The Mathematics Teacher*, *81*(6), 420-427.

Steffe, L. P., & Kieren, T. (1994). Radical constructivism and mathematics education. *Journal for Research in Mathematics Education*, *25*(6), 711-733.

Usiskin, Z. (1995). Why is algebra important to learn. *American Educator*, *19*(1), 30-37.

Wagner, D. (2017). Reflections on research positioning: Where the math is and where the people are. In H. Straehler-Pohl, N. Bohlmann, & A. Pais (Eds.), *The disorder of mathematics education: Challenging the sociopolitical dimensions of research* (pp. 291- 306). Springer.